W2L5-REDUCTION OF ORDER

Given that $y_1 = x$ is a solution of

$$x^2 y'' + 5xy' - 5y = 0$$

find a second solution and the general solution.

$y = v \cdot y_1 = vx$ ← unknown function of x

$x^2(vx)'' + 5x(vx)' - 5(vx) = 0$

$(vx)''$
$(v'x + v(1))' = v''x + v' + v'$
$= v''x + 2v'$

$x^2(v''x + 2v') + 5x(v'x + v) - 5vx = 0$

$x^3 v'' + 2x^2 v' + 5x^2 v' + \cancel{5xv} - \cancel{5xv} = 0$

$x^3 v'' + 7x^2 v' = 0$

Let $w = v'$
$w' = v''$

$x^3 \cdot w' + 7x^2 w = 0$ Solve by separation!

$x^3 \frac{dw}{dx} + 7x^2 w = 0$

$x^3 \frac{dw}{dx} = -7x^2 w$

$\int \frac{1}{w}\, dw = \int \frac{-7}{x}\, dx$

$\ln|w| = -7\ln x + C_0$

$e \qquad e$

$w = (e^{\ln x})^{-7} + e^{C_0}$

$C_1 = e^{C_0}$

$w = C_1 x^{-7}$

$v' = C_1 x^{-7}$

anti differentiate

$v = \frac{C_1 x^{-6}}{-6} + C_2$ $\qquad \tilde{C}_1 = \frac{C_1}{-6}$

$v = \tilde{C}_1 x^{-6} + C_2$

$y = v y_1 = v \cdot x$

$y = (\tilde{C}_1 x^{-6} + C_2)\, x$

$\boxed{= \tilde{C}_1 x^{-5} + C_2 x}$ ← General Solution

Second solution: x^{-5}